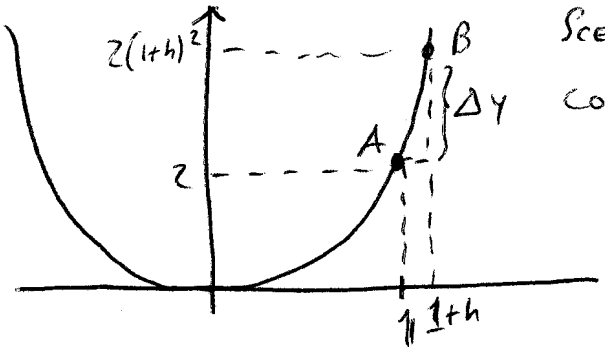
ESERCIZI SULLE DERIVATE

1) CALCOLARE LA DERIVATA (CIOÉ LA "PENDENZA") DELLA FUNZIONE $y = 2x^2$ NEL PUNTO $A(1, 2)$

SOLUZIONE



SCELGO UN PUNTO B DI COORDINATE

$B(1+h,\ 2(1+h)^2)$

LA PENDENZA DEL SEGMENTO $\overline{AB}$ É

$$\delta_{AB} = \frac{\Delta y}{h} = \frac{2(1+h)^2 - 2}{h} = \frac{2(1+h^2+2h)-2}{h}$$

$$= \frac{2+2h^2+4h-2}{h} = \frac{2h^2+4h}{h} = \frac{h(2h+4)}{h}$$

$$= 2h+4$$

LA DERIVATA "IN A" É ALLORA

$$\delta_A = \lim_{B \to A} \delta_{AB} = \lim_{h \to 0} (2h+4) = 4$$

2) CALCOLARE LA DERIVATA IN $A(\frac{1}{2}, \frac{1}{2})$

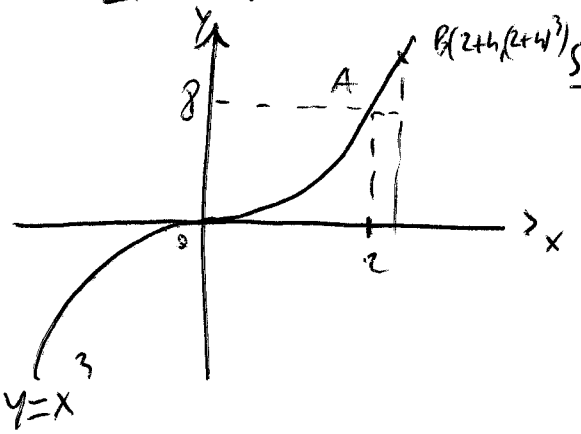
$B(\frac{1}{2}+h,\ 2(\frac{1}{2}+h)^2)$

$$\delta_{AB} = \frac{2(\frac{1}{2}+h)^2 - \frac{1}{2}}{h} = \frac{2(\frac{1}{4}+h^2+h)-\frac{1}{2}}{h} = \frac{\frac{1}{2}+2h^2+2h-\frac{1}{2}}{h}$$

$$= \frac{h(2h+2)}{h} = 2h+2$$

$$\delta_A = \lim_{B \to A} \delta_{AB} = \lim_{h \to 0} 2h+2 = 2$$

3) DATA LA FUNZIONE $y = x^3$ CALCOLARE LA DERIVATA IN $x = 2$.



SOL. $A(2, 8)$ $B(2+h, (2+h)^3)$

$$\delta_{AB} = \frac{(2+h)^3 - 8}{h} =$$

$$= \frac{8 + h^3 + 12h + 6h^2 - 8}{h} =$$

$$= \frac{h(h^2 + 12 + 6h)}{h} =$$

$$= h^2 + 12 + 6h$$

$$\delta_A = \lim_{h \to 0} h^2 + 12 + 6h = \boxed{12}$$

4) CALCOLARLA ORA IN UN GENERICO PUNTO DI ASCISSA x

PRENDO $A(x, x^3)$, $B(x+h, (x+h)^3)$

$$\delta_{AB} = \frac{(x+h)^3 - x^3}{(x+h) - x} = \frac{x^3 + h^3 + 3x^2h + 3xh^2 - x^3}{h} =$$

$$= \frac{h(h^2 + 3x^2 + 3xh)}{h} = h^2 + 3x^2 + 3xh$$

$$\delta_A = \lim_{h \to 0} (h^2 + 3x^2 + 3xh) = 3x^2$$

CIOÉ BASTA CONOSCERE LA x E AUTOMATICAMENTE SO CALCOLARE LA DERIVATA